MTH 1450 Chapter 7 Review

1 Law of Cosines and Law of Sines

Tool: The law of cosines is a generalization of Pythagrean Theorem for non right triangles

$c^2 = a^2 + b^2 - (2ab)\cos(C)$ The added term is twice the product of sides a and b multiplied by the cosine of their included angle C, which is opposite side c. NOTE: in this formulation, c need not be the longest side.

a. We may solve for an unknown triangle side if we know the other two sides and the angle between them

b. We may solve for an unknown angle if we know all three sides of a triangle.

Tool: Law of sines is useful for problems that the law of cosines can't handle

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

where a, b, c are sides of a triangle and A, B, C are their respective opposite angles.

a. Use if have two angles and a side of a triangle

b. Use if have two sides and an angle, but the angle is not included between the sides (see p. 494 for different possiblities).

Practice Problems

Observation:

What happens when angle C is 90°?

Solving for an unknown side:

Given a triangle with sides of length 1 and 3 and the angle between them is 120°. Find the length of the third side.

Ans: 4

Solving for an unknown angle:

Given a triangle with sides of length 2,3,4. Find the angles (degrees). (Hint: you may want to apply the law of cosines for each angle and check the sum)

Ans: 28.96°, 46.57°, 104.48°

Solving with two angles and a side:

We are given a triangle with side of length 5. The angles on either side are 30° and 70° respectively. Find the other two sides to 2 decimal places.

Ans: First find the remaining angle 80°. Then apply the ratios to find side opposite 30° has length 2.54 and side opposite 70° has length 4.77.

Solving with two sides and an angle not between them:

We are given a triangle with angle A of 45°, and side b adjacent to angle A has length 4. Solve for angle B opposite b given that side a opposite angle A :

a) has length 2
  Ans: no solution

b) has length 3
  Ans: two possibilities: 70.53° or 109.47°

c) has length 5
  Ans: 34.45°

2 Introduction to Polar Coordinates

Tool: With polar coordinates, we measure angle θ and radius r. We allow for negative radius by rotating by π radians or 180°. Coordinates are recorded in the form (r,θ).

Tool: When converting between rectangular and polar coordinates, it is often helpful to use a reference triangle.
To convert points from polar coordinates to rectangular, use

$$x = r\cos(\theta)$$
$$y = r\sin(\theta)$$

To convert points from rectangular to polar use

$$r = \sqrt{x^2 + y^2}$$
$$\frac{y}{x} = \tan(\theta)$$

Warning: Don' t just blindly use $\tan^{-1}$ to find θ. Remember from solving trig equations that there are multiple solutions to the equation $\tan(\theta) = \frac{y}{x}$. Use the given information to decide which quadrant your solution is in and use a reference triangle.

Tool: Converting equations between Cartesian and polar form means applying transforms and simplifying.

Plotting points:

Sketch the positions of (1, 45°) , (-2,45°), $(3, \frac{7\pi}{4})$

Converting polar to rectangular.

Convert $(2, 210°)$ and $\left(4, -\frac{\pi}{4}\right)$ to rectangular.

Ans: $(-\sqrt{3}, -1)$ and $(2\sqrt{2}, -2\sqrt{2})$

Converting rectangular to polar.

Convert $(1,1)$ and $(-1,3)$ to polar where $0 \le \theta < 360°$.

Ans: $(\sqrt{2}, 45°)$ and $(\sqrt{10}, 108.43°)$.

Convert equation from rectangular to polar.

Graph the following and convert to polar form.

$$(x-3)^2 + (y-4)^2 = 25$$

Convert equation from polar to rectangular.

$$\theta = \frac{\pi}{6}$$