MTH 1450 Chapter 6 Review

1 Fundamental Identities

Tool: Identities are true for all values of angle θ and are grouped into categories. They may be used to simplify expressions.

Reciprocal Identities:

$$csc(\theta)=\frac{1}{\sin(\theta)},\quad sec(\theta)=\frac{1}{\cos(\theta)},$$
$$\cot(\theta)=\frac{1}{\tan(\theta)}$$

Quotient Identities:

$$\tan(\theta)=\frac{\sin(\theta)}{\cos(\theta)},\quad \cot(\theta)=\frac{\cos(\theta)}{\sin(\theta)}$$

Even-Odd Identities:

$$\sin(-\theta)=-\sin(\theta),\quad \cos(-\theta)=\cos(\theta),$$
$$csc(-\theta)=-csc(\theta),\quad sec(-\theta)=sec(\theta),$$
$$\tan(-\theta)=-\tan(\theta),\quad \cot(-\theta)=-\cot(\theta)$$

Pythagorean Identities:

$$\cos^2(\theta)+\sin^2(\theta)=1,\quad 1+\tan^2(\theta)=sec^2(\theta),$$
$$1+\cot^2(\theta)=csc^2(\theta)$$

Tool: Verifying trig identities (see p. 441). An identity is an equation that should be true for all values of θ. If it is, then you can use the identities you already know to simplify both sides to be equal to each other.

Other suggestions include writing in terms of sines and cosines, combining fractions over a common denominator, factoring, and combining like terms.

Tool: Disproving false identities takes one single angle that shows they don't work. You may have to try more than one if you try this method.

2 Sum and Difference Angle Identities

Tool: Sum and difference formulas for cosine maybe proven using distance formulas on unit circle using x and y coordinates (see pp. 446-7)

$$\cos(u+v)=\cos(u)\cos(v)-\sin(u)\sin(v)$$
$$\cos(u-v)=\cos(u)\cos(v)+\sin(u)\sin(v)$$

Tool: Cofunction identities by observing that if we substitute complementary angles in triangles, we switch our point of view.

$$\sin\left(\frac{\pi}{2}-v\right)=\cos(v),\quad \cos\left(\frac{\pi}{2}-v\right)=\sin(v)$$
$$csc\left(\frac{\pi}{2}-v\right)=sec(v),\quad sec\left(\frac{\pi}{2}-v\right)=csc(v)$$
$$\tan\left(\frac{\pi}{2}-v\right)=\cot(v),\quad \cot\left(\frac{\pi}{2}-v\right)=\tan(v)$$

Practice Problems

Simplifying trig expressions:

$$\tan^2(x)\cos^2(x)+\cos^2(x)$$

Ans: 1

$$\cot^4(t)+\cot^2(t)+csc^2(t)$$

Ans: $csc^4(t)$

Verifying Trig identities for all angular values:

$$\sin^2(x)(1+\cot^2(x))=1$$

$$\frac{\sin(t)}{1-\cos(t)}=\frac{1+\cos(t)}{\sin(t)}$$

Proving an equation is not an identitiy:

$$\sin(x)+\cos(x)=1$$

$$\sin(u)+\sin(v)=\sin(u+v)$$

Practice with cosine sum and difference formulas:

Compute the following exactly (no decimal).

$$\cos(75)$$
$$\cos\left(\frac{\pi}{2}-t\right)$$

Cofunction identity practice:

Given $\sin(\theta)=\frac{1}{3}$

What is $sec(90-\theta)$?

Tool: Sum and difference formulas for sine are proven by cofunction identities.

$$\sin(u+v)=\sin(u)\cos(v)+\sin(u)\cos(v)$$
$$\sin(u-v)=\sin(u)\cos(v)-\sin(v)\cos(u)$$

Tool: Addition and Subtraction identities for tangent.

$$\tan(u-v)=\frac{\tan(u)-\tan(v)}{1+\tan(u)\tan(v)}$$

3 Double and Half Angle Identities

Tool: If we set $u=v=\theta$ in the sum angle formulas, we obtain the double angle formulas. Note that double angle formula for cosine has a couple of equivalent forms based on the Pythagorean identity.

$$\begin{aligned}\cos(2\theta)&=\cos^2(\theta)-\sin^2(\theta)\\&=1-2\sin^2(\theta)\\&=2\cos^2(\theta)-1\end{aligned}$$
$$\sin(2\theta)=2\sin(\theta)\cos(\theta)$$
$$\tan(2\theta)=\frac{2\tan(\theta)}{1-\tan^2(\theta)}$$

Tool: Half angle identities. These are derived from the double angle cosine identities and replacing θ by $\frac{\theta}{2}$ (see p. 462).

$$\cos\left(\frac{\theta}{2}\right)=\pm\sqrt{\frac{1+\cos(\theta)}{2}}$$
$$\sin\left(\frac{\theta}{2}\right)=\pm\sqrt{\frac{1-\cos(\theta)}{2}}$$
$$\tan\left(\frac{\theta}{2}\right)=\frac{1-\cos(\theta)}{\sin(\theta)}$$

Evaluate the expression exactly (no decimal):
Hint: recognize the proper identity to simplify

$$\sin(66)\cos(21)-\cos(66)\sin(21)$$

Evaluate the expression exactly (no decimal):
Hint: inverse trig functions output angles. Use triangles to find those angles.

$$\sin\left(\sin^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{3}{4}\right)\right)$$

(optional)Practice: You don't really need to memorize pairs of sum and difference angle formulas because you can use even-odd properties to derive one from the other.
Derive the formula for $\tan(u+v)$ using even-odd identities for tangent.

Evaluating trig functions.

Let θ be a first quadrant angle such that $\tan(\theta)=\frac{5}{12}$.

Compute exactly (no decimal).

$\sin(2\theta)$, $\cos(2\theta)$, $\tan(2\theta)$.

Inverse trig.

Evaluate exactly (no decimal) $\cos\left(2\sin^{-1}\left(\frac{2}{3}\right)\right)$.

Identity practice.
Show that

$$\cos(2\theta)-\sin(2\theta)=(\cos(\theta)-\sin(\theta))^2-2\sin^2(\theta)$$

Half angles.

If $\sin(\theta)=\frac{3}{4}$ and θ is in first quadrant, then compute

$\cos\left(\frac{\theta}{2}\right)$, $\sin\left(\frac{\theta}{2}\right)$, $\tan\left(\frac{\theta}{2}\right)$.

4 Trig Equations

Tool: Trig equations are true only for certain values of an angle (unlike identities, which are true for all values). Always be aware of restrictions. They will tell you where to look for solutions when you apply inverse trig functions.

Practice solving equations.
Solve $\tan(\theta)=1$ exactly
a) For $0 \leq \theta < 90$
b) For $0 \leq \theta < 360$
c) Unrestricted θ

Tool: Reference triangles can help you find multiple solutions of trig equations.

Practice.
Solve $\sin(\theta)=0.9563$ to two decimal places for $0 \leq \theta < 180$.
Ans: 73°, 107°

Tool: Known solutions and 30-60-90, 45-45-90 triangles can help solve.

Solve exactly (no decimal).
$\cos\left(2\theta-\frac{\pi}{6}\right)=\frac{1}{2}$ For $0 \leq \theta < \frac{\pi}{2}$

Ans: $\frac{\pi}{4}$

Tool: Factoring and finding zeros of factors may help find solutions. Think back to when you factored quadratics, but now we have trig functions.

Trig factoring.
Solve $\tan(\theta)\sin(\theta)-\tan(\theta)=0$ for $0 \leq \theta < 360$.
Ans: 0°, 90°,180°

Tool: Trig identities can be used to solve. You may have to factor.

Solve $\sin(2\theta)=\sin(\theta)$ for $0 \leq \theta < 360$
Ans: 0°, 60°,180°,300°.