MTH 1450 Chapter 2 Review

1 The Function Concept

Tool: A function is a rule of correspondence between a **domain** set of input values and **range** set of output values such that each element in the domain corresponds to exactly one element of the range.

Tool: In algebra we have a compact notation for representing the outputs of functions. For a function named f that takes inputs x we say the outputs y are given by $y=f(x)$. Note that the parentheses do not mean multiplication.

With function notation we often give a formula for computing the outputs of a function. Sometimes this formula may be given piecewise for different values of x.

Note that functions, inputs and outputs can be called any name you want. For a function named s with inputs t and outputs v, we say $v=s(t)$.

Tool: Some functions have naturally restricted domains because division by zero is not allowed. Also in this class all our outputs are real, so we do not allow even roots of negative numbers.

If a function does not have a restriction on inputs, we say its domain is all real numbers or $(-\infty,\infty)$. Examples of such functions include lines and polynomials.

Tool: Difference quotients are ways to compute approximate slopes of functions, even if they are not lines. If $y=f(x)$, then the slope between (x_1, y_1) and (x_2, y_2) may be written as $m=\frac{y_2-y_1}{x_2-x_1}$, but because the y values are function outputs we can write everything in terms of x values, so $m=\frac{f(x_2)-f(x_1)}{x_2-x_1}$. If we let $x_1=t$ vary and $x_2=t+h$, then we obtain a new function called the difference quotient of f. **IMPORTANT FORMULA**

$$m=\frac{f(t+h)-f(t)}{h}$$

Difference quotients are often used to find average rates of change, such as speeds.

Practice Problems

Function identification:
One way to represent functions is by tables of values. Below are three tables put side by side with x representing inputs and y representing outputs. See if you can determine which is a function.

x	y	***	x	y	***	x	y
1	-115	***	1	115	***	1	115
115	0	***	1	115	***	1	0
0	115	***	0	115	***	0	115

Outputs of a function:

If $f(x)=x^3-2$, compute
$f(2)$, $f(-2)$, $\sqrt{f(3)}$

If $g(x)=-x+3$, compute
$g(-3t)$, $g(t-1)$, $g(t)-1$

If $h(x)=\begin{cases} 1, & \text{if } x<0 \\ 15, & \text{if } x=0 \\ x+1, & \text{if } x>0 \end{cases}$, compute
$h(-5)$, $h(0)$, $h(5)$

Find the domains:

$f(x)=x^4-1$

$g(y)=\frac{y+2}{y(y^2-1)}$

$w(z)=\sqrt{1-z}$

Computing difference quotients:

$f(x)=\frac{3}{2}x-14$

$g(x)=1-x^2$

Application of difference quotient:
A cannonball falls from a height of 64 feet with position x at time t given by $x(t)=-16t^2+64$. Compute the difference quotient. Then evaluate numerically for

$t=2, h=1$
$t=2, h=0.1$
$t=2, h=0.01$
$t=2, h=0.001$

. What do the resulting numbers mean?

2 Graphs of functions

Tool: Vertical line test. A graph represents a function if and only if every vertical line intersects it in at most one point. Graphs that are not functions can be c2hanged into functions by taking an appropriate piece.

Vertical line test:
Graph the following. Which of the following graphs are functions? If they are not functions, how can we decompose them into functions?

$y=2x+3$

$y=x^2$

$x=y^2$

$x^2+y^2=1$

Tool: Graphs may be used to determine domain and range of a function.

Domain and range: Graph the following and determine domain and range

$y=2x+3$

$y=x^2$

$y=\sqrt{x}$

$y=-\sqrt{1-x^2}$

Application: (see # 31, p. 153)

3 Graph Features

Tool: Sign of a function. To solve $f(x)>0$, check where the graph lies above the x-axis and mark those values as a set. Inversely, to solve $f(x)<0$, check where the graph lies below the x-axis.

Sign of function.
Solve $f(x)=3-|x-3|\geq 0$.
Ans: $[0,6]$

Tool: Determining increasing or decreasing behavior. Try to visualize the graph in terms of slopes of lines. If the slopes are positive, then the graph is increasing. If the slopes are negative, graph is decreasing. If the slope is zero, the graph is constant.

Increasing or decreasing behavior.
Graph the function

$$h(x)=\begin{cases}-x^2+1, & \text{if } x<0\\ 1, & \text{if } 0\leq x<2\\ x-1, & \text{if } 2\leq x<5\\ 9-x, & \text{if } x\geq 5\end{cases}$$

and indicate where it is increasing, decreasing or constant.

Tool: If f is an even function then it is symmetric about y-axis and $f(-x)=f(x)$. If f is an odd function then it is symmetric about the origin and $f(-x)=-f(x)$. Examples of even functions are even powers of x, and of odd functions are odd powers of x.

Determine if the following functions are even, odd or neither.

$g(x)=-x^3+x$

$f(x)=x^4+|x|+1$

$w(x)=|x|+x$

Tool: Standard graphs. Know the eight types of graph on p. 162: linear, constant, square, square root, absolute value, cube, cube root, semicircle. You need to know shape, domain, range, increasing/decreasing intervals, and symmetry. (If you know the shape of the graph, you can read the other properties from it).

Piecewise graph application.
Suppose average price of a gallon of milk is $2 for a couple of years. Then the the price changes via $0.5t+2$ for three more years (t in years). Then it changes again via $9-t$ for two more years. What are the highest and lowest prices of the milk? Which changes are discontinuous?

4 Transformations of Graphs

Tool: Vertical shift of $f(x)$ occurs when we modify the output to obtain $f(x)+k$. Up shift if $k>0$ and down shift if $k<0$.

Sketch graphs of $|x|+2$ and $|x|-2$
Note the difference.

Tool: Horizontal shift of $f(x)$ occurs when we modify the output to obtain $f(x-h)$. Right shift if $h>0$. Left shift if $h<0$.

Sketch graphs of $(x+2)^3$ and $(x-2)^3$
Note the difference.

Tool: Reflection about y -axis occurs when we replace $f(x)$ by $-f(x)$. Reflection about x -axis occurs when we replace $f(x)$ by $f(-x)$.

Sketch graphs of $-\sqrt{x}$ and $\sqrt{-x}$.
Note the difference.

Tool: Vertical scaling of $f(x)$ occurs when we replace $f(x)$ by $af(x)$.

Sketch graphs of $\frac{1}{2}x^2$ and $2x^2$.

Tool: Multiple transformations. Combinations of shifts, reflections, scalings, may be treated by sketching a base graph and proceeding step by step, like a recipe. These operations give a window into how computer graphics technology works.

Sketch graphs of $-2(x+3)^2-4$ and $\sqrt[3]{1-x}+2$
You may want to double check your transformations by plotting a couple points.

Application: p. 184 #60.

5 Combinations of Functions

Tool: Sum, difference, product, quotient functions. If we do operations to the outputs of functions we can obtain new functions.

$(f+g)(x)=f(x)+g(x)$
$(f-g)(x)=f(x)-g(x)$
$(f\cdot g)(x)=f(x)\cdot g(x)$
$(f/g)(x)=f(x)/g(x)$

Practice. Let $f(x)=x-2$, $g(x)=x^2-2$
What do you get when you add a line and a parabola?
Compute and find domains of
$h(x)=(f+g)(x)$
$(f-g)(x)$
$(f\cdot g)(x)$
$\left(\frac{f}{g}\right)(x)$

Tool: Composition is when you take the output of one function as the input of another function. Just plug in the values.

$(f\circ g)(x)=f(g(x))$

Practice. Let $f(x)=\sqrt{x}$ and $g(x)=x-2$
Compute
$(f\circ g)(2)$, $(g\circ f)(2)$, $(g\circ g)(2)$
$(f\circ g)(x)$, $(g\circ f)(x)$, $(f\circ f)(x)$

Let $h(x)=\sqrt[3]{x}$ and $k(x)=x^3$
Compute
$(f\circ g)(x)$
$(g\circ f)(x)$

Let $m(x)=(3\text{x}+4)^3$ and decompose into the composition of two functions.

Application: Suppose you sell a commodity and want to maximize profit over time. Producer price functions for low and high refining capacities respectively are $a(x)=x$, and $b(x)=\sqrt[3]{x}$. Consumer functions for high and low demands respectively are $c(t)=t$ and $d(t)=\sqrt{t}$. Compute $(a\circ c)(t)$, $(a\circ d)(t)$, $(b\circ c)(t)$, $(b\circ d)(t)$ to project growth in prices.