

MTH 1450 Chapter 1 Review

1 Linear Equations

Tool: We can add, subtract, multiply and divide both sides of an equation by the same quantity (except no division by zero). So we can simplify equations to solve for variables.

If $a = b$, then (assuming no zeros in denominators)

$$a + c = b + c$$

$$a - c = b - c$$

$$a \cdot c = b \cdot c$$

$$a/c = b/c$$

$$1/a = 1/b$$

Tool: Absolute value equations may be split into two linear equations without absolute value bars. The equation

$$|ax + b| = k$$

means

$$ax + b = k \text{ or } ax + b = -k$$

Tool: In word problems there is no recipe. Look at the question to find the variable(s) you need. Put together with any given (or implied) numbers and see if the equation makes sense. Look at the different types of word problems in the text to gain a feel for what's out there.

2 Nonlinear equations

Tool: Group terms on one side of equation and factor. If

$$(x - r)(x - s) = 0, \text{ then } x = r \text{ or } x = s$$

Tool: Isolate square on one side and take square root of both sides. The equation $(ax + b)^2 = k$ is equivalent to the pair of equations

$$ax + b = \sqrt{k}$$

$$ax + b = -\sqrt{k}$$

Tool: Completing the square (see p. 54 for alternate version).

Use for quadratic equations $ax^2 + bx + c = 0$ too hard to factor. Subtract c from both sides and divide through by a

to get $x^2 + \frac{b}{a}x = \frac{-c}{a}$. Then add $\left(\frac{b}{2a}\right)^2$ to both sides and

factor to obtain $\left(x + \frac{b}{2a}\right)^2 = \frac{-c}{a} - \left(\frac{b}{2a}\right)^2$, which you can square root.

Tool: Quadratic formula gives equivalent solution to completing the square. Use when coefficients are nasty.

For $ax^2 + bx + c = 0$ we have two solutions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Practice Problems

Basic Equation: (solve for s)

$$\frac{5}{4}s - 1 = \frac{5}{4}$$

Rational Equation: (solve for r, check extraneous solution)

$$\frac{1}{r-2} - \frac{1}{r} = \frac{1}{15r}$$

Formulas: (solve for h)

$$A = \frac{1}{2}\left(a + \frac{b}{h}\right)$$

Absolute Value Equations: (solve for x)

$$|x| = -3$$

$$|2x - 1| = 2x + 1$$

Linear Word problem:

A gambler plays the stock market. He bets \$10000 spread between a low risk stock returning 5% and a high risk stock returning 10%. Assuming no crashes, total income for the year is \$850. How much did he invest in each stock?

Factor problem: (solve for w)

$$w = w^2 + 12$$

Square root problem: (solve for y)

Recall that $\sqrt{-1} \equiv i$.

$$(3y + 4)^2 + 4 = 0$$

Completing the square problem: (solve for x)

$$x^2 - 4x + 5 = 0$$

Polynomial factoring: (solve for x, four solutions)

$$x^4 - 16 = 0$$

Rational quadratic: (solve for v, check extraneous solution)

$$\frac{2}{v-2} - \frac{2}{v+2} = 1$$

Quadratic formula problem: (solve for t)

$$1.21t^2 - 3.54t + 7.29 = 0$$

Ans: $t = 1.463 \pm 1.971i$

Tool: Solving equations with roots you solve by raising both sides to the appropriate power and simplifying/solving.

Recall that $(\sqrt{x})^2 = x$ and $(\sqrt[n]{x})^n = x$.

Note that in some problems you may have to square both sides more than once to remove the roots.

3 Inequalities

Tool: Just like for equations, we can add, subtract, multiply and divide both sides of an inequality by the same quantity (except no division by zero). The only catch is that if you multiply or divide by a negative quantity, you must switch the sign (this means no multiplying through by variables, because you don't know if they are positive or negative).

Tool: Absolute value inequalities may be transformed into linear inequalities as follows. (It helps to think of absolute value as denoting size or distance.) If $|x| < k$, then x is no larger than k in magnitude and $-k < x < k$.

Alternatively, if we switch to $|x| > k$, then x lies outside of a bounded interval, so $x < -k$ or $x > k$.

Tool: Nonlinear inequalities that are factorable may use the cut-point method (p. 68). In summary you look at each factor and its zero on the number line. Decide whether each zero is a solution. The zeros mark out intervals on the number line. Pick a test point on the interior of each interval. If the test point solves the inequality, mark the interval as part of the solution.

4 Cartesian Coordinate System

Tool: Plotting points (x,y) is like playing battleships only you are using two real number lines named x and y .

Tool: Distance formula. When you have two points (x_1, y_1) and (x_2, y_2) you can algebraically find the distance between them to great accuracy using

$$d = \sqrt{((x_1 - x_2)^2 + (y_1 - y_2)^2)}$$

This formula is just another way of stating the Pythagorean theorem $a^2 + b^2 = c^2$

Tool: Midpoint formula. You can find a point that's right in between two points (x_1, y_1) and (x_2, y_2) . Just take the arithmetic average of coordinates

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$

Cube root equation: (solve for x)

$$\sqrt[3]{3x-1} + 1 = 0$$

Square root equation: (solve for y)

$$\sqrt{3y+3} - \sqrt{2y} = 1$$

Geometry problem: A triangle has height 2 greater than its base. If the area of the triangle is 12, find the dimensions.

Linear inequalities: (solve for x and graph)

$$-2x + 1 < -3$$

$$-2 \leq \frac{-5}{2}x + 1 < 2$$

Absolute value inequalities: (solve for y and graph)

$$\left| \frac{2}{3}x - 1 \right| \geq 1$$

$$-|1 - x| > -2$$

Nonlinear inequalities: (solve for z and graph)

$$z^3 - 16z \leq 0$$

$$\frac{1}{z-1} > \frac{1}{z}$$

Inequality word problem:

The gambling investor is on the run from the IRS for not paying his taxes. Suppose from Detroit to Kansas City, the price of gas was \$3.00 per gallon, and from Kansas city to Los Angeles the price of gas was \$4.00 per gallon. Assuming he spent \$470 total needed 130 gallons for his trip (2600 miles @ 20 mpg), what's the least he had to pay for gas from Detroit to Kansas City? (Hint: let x be the amount paid for gas between Detroit and Kansas city.)

Ans: $x \geq \$150$

Plotting points:

Practice plotting $(-2,6)$, $(0,-4)$, $(3,0)$ on a pair of axes.

Finding distances:

between $(0,-4)$ and $(3,0)$

Ans: 5

between $(-2.25,6.78)$ and $(3.45,-10.65)$

Ans: 18.338

Finding midpoint:

between $(14,-17)$ and $(-4,10)$

between $(t, 1-t)$ and $(2-3t, 1+t)$

Tool: Graphing equations by plotting points. If you have equations with x and y in them, you can set one variable and solve for the other. For instance if you set $x=0$ and solve for y you obtain a point called the y -intercept. Set $y=0$ to find the x -intercept.

Tool: Nonlinear equations can use point plotting as well, but because they curve, it's often useful to use symmetry to find out where they go.

y -axis symmetry: replacing x by $-x$ gives same equation. (even powers or absolute value of x)

x -axis symmetry: replacing y by $-y$ gives same equation. (even powers or absolute value of y)

origin symmetry: x by $-x$ and y by $-y$ gives same equation. (odd powers of x and y)

Tool: Technology. Calculators and computers can plot points far faster than we can and they don't even get bored. If you plan to go into science, engineering, or business, they are worth the investment. Unfortunately only calculators are allowed for exams.

Tool: Variation can be used to look at a relation between quantities. We express in terms of formulas which can contain variables, which change, or variables which do not.

5 Lines and Circles

Tool: Slope formula. Given points (x_1, y_1) and (x_2, y_2) , the slope of the line through them is $m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$.

Positive slopes rise to the right. Negative slopes fall to the right.

Tool: Point-slope form of a line. Given a point (x_1, y_1) and slope m , an equation that gives the rest of the points on the line is $y - y_1 = m(x - x_1)$. Note the similarity to the slope formula (the rest of the points give the same slope relative to (x_1, y_1)).

Graphing lines in the Cartesian Plane:

graph $y = -3$

graph $x = -2$

graph $y = -x + 2$

Plotting nonlinear graphs:

$y = 2 - x^2$

$x = 2 - y^2$

$y = x^3 + 2x$

Use your favorite calculator/computer program:

plot $x^3 - 5x^2 + x$
xmin -10; xmax 10; xscl 1
ymin -30; ymax 30; yscl 1

plot $x^4 - 8x^2 - 9$
xmin -4; xmax 4; xscl 1
ymin -30; ymax 30; yscl 1

plot $x^5 - 5x^4 + 6x^3 + 2x^2 - 5x$
xmin -10; xmax 10; xscl 1
ymin -5; ymax 5; yscl 1

Inverse variation:

Assuming distance is constant k as in a series of races, we may say the speed s of each racer is inversely proportional to the time t they require to complete the race, or $s = \frac{k}{t}$. If

a runner going 8 meter per second completes the race in 50 seconds, how long is the track? Can you make a plot of race times vs. speeds?

Slope problem:

Compute the slope of the line through $(0, 4)$ and $(-2, -3)$.

Point-slope form:

Write the equation of the line through $(15.4, 3.2)$ and $(7.4, 1.2)$ in point-slope form.

Word problem: (pessimistic prediction)

A car bought for \$16,000 is worth \$9,000 in two years. Find the linear equation describing the depreciation and predict when the car will be worthless.

Tool: Slope intercept form of a line: $y = mx + b$ where m is the slope and $(0, b)$ is the y -intercept. Really it's just point-slope form where (x_1, y_1) is the y -intercept. You can convert from point-slope to slope-intercept form by solving for y .

Tool: Slopes of parallel lines are equal, $m_1 = m_2$. Slopes of perpendicular lines are negative reciprocal $m_1 = \frac{-1}{m_2}$

Tool: Standard equation of a circle with center (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$. Equations containing x^2 and y^2 may need completing the square to write in this form.

Slope-intercept form:

Write the equation of the line through $(3, 6)$ and $(1, 2)$ in slope-intercept form.

Parallel and perpendicular lines:

For the line $x + y = 1$, find the equation of

a) the parallel line through $(3, 2)$

b) the perpendicular line through $(3, 2)$

Circles:

Write the equation of the circle with radius 4 going through $(-1, 2)$.

Write in standard form of circle and graph

$$x^2 - 4x + y^2 + 6y + 4 = 0$$