

Supplement 2

DIFFERENTIATION OF e^x AND $\ln x$

$$\frac{d}{dx}(e^x) = e^x \quad (2.1)$$

Example 1. Find $\frac{d}{dx}(e^{2x})$. Since $e^{2x} = e^x e^x$, we may proceed as follows:

$$\frac{d}{dx}(e^{2x}) = \frac{d}{dx}(e^x e^x) = e^x \frac{d}{dx}(e^x) + e^x \frac{d}{dx}(e^x) = e^x e^x + e^x e^x = e^{2x} + e^{2x} = 2e^{2x}$$

Example 2. Use the result of Example 1 to find $\frac{d}{dx}(e^{3x})$

$$\frac{d}{dx}(e^{3x}) = \frac{d}{dx}(e^{2x} e^x) = e^{2x} \frac{d}{dx}(e^x) + e^x \frac{d}{dx}(e^{2x}) = e^{2x} e^x + e^x (2e^{2x}) = e^{3x} + 2e^{3x} = 3e^{3x}$$

Example 3. Differentiate $f(x) = e^{x+3}$

$$f'(x) = \frac{d}{dx}(e^{x+3}) = \frac{d}{dx}(e^x e^3) = e^3 \frac{d}{dx}(e^x) = e^3 e^x = e^{x+3}$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x} \quad (x > 0) \quad (2.2)$$

Example 3. Find $\frac{d}{dx}(\ln x^3)$. Since $\ln x^3 = 3 \ln x$, we may proceed as follows:

$$\frac{d}{dx}(\ln x^3) = \frac{d}{dx}(3 \ln x) = 3 \frac{d}{dx}(\ln x) = 3 \left(\frac{1}{x} \right) = \frac{3}{x}$$

Example 4. Differentiate $f(x) = e^x + \ln x + 3x^5$

$$f'(x) = \frac{d}{dx}(e^x) + \frac{d}{dx}(\ln x) + \frac{d}{dx}(3x^5) = e^x + \frac{1}{x} + 15x^4$$

Example 5. Differentiate $f(x) = \ln(3x)$. Since $\ln(3x) = \ln 3 + \ln x$,

$$\frac{d}{dx}(\ln 3x) = \frac{d}{dx}(\ln 3) + \frac{d}{dx}(\ln x) = 0 + \frac{1}{x} = \frac{1}{x}$$

PROBLEMS: Find $\frac{dy}{dx}$ if:

1. $y = e^{4x}$	Ans. $4e^{4x}$
2. $y = \ln 4x$	Ans. $\frac{1}{x}$
3. $y = e^x \sin x$	Ans. $e^x \sin x + e^x \cos x$
4. $y = \frac{e^x}{\cos x}$	Ans. $\frac{e^x \cos x + e^x \sin x}{\cos^2 x}$
5. $y = e^x \ln x$	Ans. $e^x \ln x + \frac{e^x}{x}$
6. $y = x^3 e^x$	Ans. $3x^2 e^x + x^3 e^x$
7. $y = x^4 \ln x$	Ans. $x^3 + 4x^3 \ln x$
8. $y = 3x^5 + \tan x - \ln x + 4e^x$	Ans. $15x^4 + \sec^2 x - \frac{1}{x} + 4e^x$
9. $y = \ln\left(\frac{x}{3}\right)$	Ans. $\frac{1}{x}$
10. $y = \frac{\ln x}{e^x}$	Ans. $\frac{1 - x \ln x}{xe^x}$

Supplement 3

Additional Antiderivatives

Function	Particular antiderivative
e^x	e^x (3.1)
$\frac{1}{x}$	$\ln x$ if $x > 0$ (3.2)

Example 1. Find the most general antiderivative of $f(x) = x^2 + 3e^x$

$$F(x) = \frac{1}{3}x^3 + 3e^x + c$$

Example 2. Find the most general antiderivative of $f(x) = \frac{x^4 + 2}{x}$

$$\text{Since } f(x) = \frac{x^4 + 2}{x} = x^3 + \frac{2}{x}, F(x) = \frac{1}{4}x^4 + 2\ln x + c \text{ if } x > 0$$

PROBLEMS: Find the most general antiderivative of:

1. $f(x) = e^x + 4x^3 + 2$	Ans. $F(x) = e^x + x^4 + 2x + c$
2. $f(x) = \frac{5}{x}$	Ans. $F(x) = 5\ln x$ if $x > 0$
3. $f(x) = 2e^x - \sin x$	Ans. $F(x) = 2e^x + \cos x + c$
4. $f(x) = \frac{1}{5x}$	Ans. $F(x) = \frac{1}{5}\ln x$ if $x > 0$

Supplement 4

Since $\frac{d}{dx}(e^x) = e^x$ (2.1)	Then $\int e^x dx = e^x + c$ (4.1)
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And, since

$$\ln|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases} \quad (4.2)$$

then

$$\frac{d}{dx} \ln|x| = \frac{d}{dx} \ln x = \frac{1}{x} \text{ if } x > 0 \quad (4.3)$$

And, by the chain rule,

$$\frac{d}{dx} \ln|x| = \frac{d}{dx} \ln(-x) = \frac{1}{-x}(-1) = \frac{1}{x} \text{ if } x < 0 \quad (4.4)$$

Hence, $\frac{d}{dx} \ln x = \frac{1}{x}$ if $x \neq 0$ (4.5)	So, $\int \frac{1}{x} dx = \ln x + c$ if $x \neq 0$ (4.6)
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Example 1. $\int (x^2 + 3e^x) dx = \frac{1}{3}x^3 + 3e^x + c$

Example 2. $\int \frac{x^4 + 2}{x} dx = \int \left(x^3 + \frac{2}{x} \right) dx = \frac{1}{4}x^4 + 2\ln|x| + c$ if $x \neq 0$

Find the following indefinite integrals:

1. $\int (e^x + 4x^3 + 2) dx =$	Ans. $e^x + x^4 + 2x + c$
2. $\int \frac{5}{x} dx =$	Ans. $5\ln x $ if $x \neq 0$
3. $\int (2e^x - \sin x) dx =$	Ans. $2e^x + \cos x + c$
4. $\int \frac{1}{5x} dx =$	Ans. $\frac{1}{5}\ln x $ if $x \neq 0$